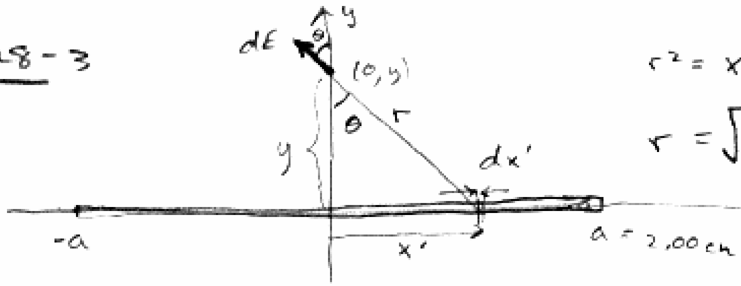
SAC 428-3

a)



$r^2 = x'^2 + y^2$

$r = \sqrt{x'^2 + y^2}$

$L = a - -a = 2a = 2(2.00\,cm)$

$L = 4.00\,cm = .0400\,m$

Given that the total charge Q is uniformly distributed on the line segment of length L, we have

$$\lambda = \frac{Q}{L} = \frac{5.60\times10^{-7}c}{.0400\,m}$$

$$\lambda = 1.40\times10^{-5}\,\frac{c}{m} \text{ (constant)}$$

By symmetry, $E_x = 0$. More specifically, for every bit of charge to the right of the y axis creating an electric field at $(0,y)$ with a leftward x component dE_x, there is a bit of charge to the left of the y axis creating an electric field at $(0,y)$ with a rightward x component of the same magnitude dE_x. Thus, all the contributions to E_x cancel pairwise.

So all we have to do is find E_y.

From Coulomb's Law:

$$dE = \frac{k\,dq}{r^2}$$

$$dE = \frac{k\lambda\,dx'}{r^2}$$

Need the y-comp. of dE:

$dE_y = dE\cos\theta$

From ( ) we see that

$$\cos\theta = \frac{y}{r}$$

so $dE_y = dE\,\frac{y}{r}$

Using $dE = \frac{k\lambda dx'}{r}$ from above we find:

$$dE_y = \frac{k\lambda dx'}{r^2}\,\frac{y}{r}$$

$$dE_y = k\lambda y\,\frac{dx'}{r^3}$$

Subst'g $r = \sqrt{x'^2+y^2}$ yields

$$dE_y = k\lambda y\,\frac{dx'}{\left(\sqrt{x'^2+y^2}\right)^3}$$

$$dE_y = k\lambda y\,\frac{dx'}{(x'^2+y^2)^{3/2}}$$

Integrating over the segment yields

$$\int dE_y = k\lambda y\int_{-a}^{a}\frac{dx'}{(x'^2+y^2)^{3/2}}$$

$$E_y = k\lambda y\,\frac{1}{y^2}\,\frac{x'}{\sqrt{x'^2+y^2}}\Bigg|_{-a}^{a}$$

$$E_y = \frac{k\lambda}{y}\left[\frac{a}{\sqrt{a^2+y^2}} - \frac{-a}{\sqrt{(-a)^2+y^2}}\right]$$

$$E_y = \frac{2k\lambda a}{y\sqrt{y^2+a^2}}$$

Evaluating at given/known values yields

$$E_y = \frac{2\left(8.99\times10^9\,\frac{N\,m^2}{c^2}\right)\left(1.40\times10^{-5}\,\frac{c}{m}\right)(.0200\,m)}{.0500\,m\sqrt{(.0500\,m)^2+(.0200\,m)^2}}$$

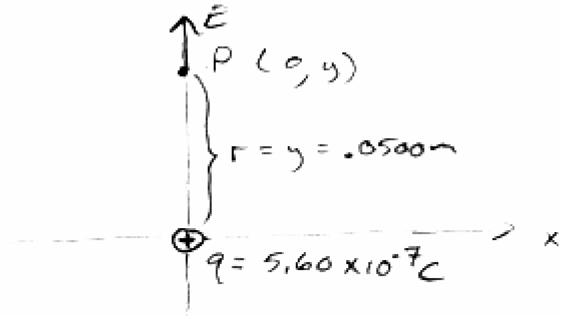
$$E_y = 1.87\times10^6\,\frac{N}{C}$$

$$\boxed{\vec{E} = 1.87\times10^6\,\frac{N}{C}\ \updownarrow}$$

(CONT'D)

SAC 4 28-3 (CONT'D)

b)



$$E = \frac{kq}{r^2} = \frac{8.99 \times 10^9 \frac{N \cdot m^2}{C^2}(5.60 \times 10^{-7} C)}{(.0500m)^2}$$

$$E = 2.01 \times 10^6 \frac{N}{C}$$

The magnitude of the electric field due the point charge is of the same order of magnitude but <u>greater</u> than the magnitude of the electric field due to the line of charge.